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Limit Cycles to Enhance Human Performance Based on Phase Oscillators

Wearable robots including exoskeletons, powered prosthetics, and powered orthotics must add energy to the person at an appropriate time to enhance, augment, or supplement human performance. This "energy pumping" at resonance can reduce the metabolic cost of performing cyclic tasks. Many human tasks such as walking, running, and hopping are repeating or cyclic tasks where assistance is needed at a repeating rate at the correct time. By utilizing resonant energy pumping, a tiny amount of energy is added at an appropriate time that results in an amplified response. However, when the system dynamics is varying or uncertain, resonant boundaries are not clearly defined. We have developed a method to add energy at resonance so the system attains the limit cycle based on a phase oscillator. The oscillator is robust to disturbances and initial conditions and allows our robots to enhance running, reduce metabolic cost, and increase hop height. These methods are general and can be used in other areas such as energy harvesting.
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Introduction

Wearable robotic systems can enhance and assist a user in a variety of tasks such as walking, running, and hopping. Technological developments have allowed miniaturization of sensors, microprocessors, and an increase in power density of batteries,

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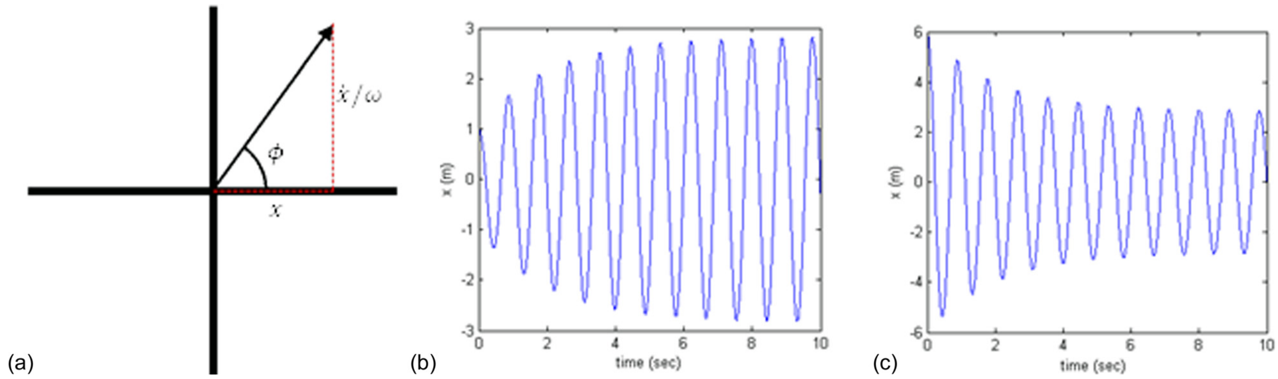


Fig. 1 (*a*) ϕ, shown on a phase plot, is defined here as *a*tan2($\dot{x}/\omega, x$). (*b*) Spring response with phase oscillator. $m = 1$ kg, $b = 1$ Ns/m, $k = 50$ N/m, $c = 20$ N, initial position = 1 m, and initial velocity = 0 m/s. (*c*) initial position = 6 m and initial velocity = 0 m/s.

but despite these advances, many wearable systems have not been able to assist a user because of the added weight burden of carrying the robotic device and the difficulty of assisting a user at the appropriate time. Only a few systems have shown metabolic savings [1–3].

A controller based on a phase oscillator is able to pump energy into the gait cycle at resonance to assist the user and achieve metabolic savings. The controller is robust to initial conditions and disturbances. It has been used to enhance hop height [4], reduce the metabolic cost while running [2], and modulate the force to create a hopping robot [5]. The phase oscillator is a general controller that can be used in other areas such as energy harvesting.

A linear resonance is defined when a system is forced at its natural frequency resulting in an amplified response, for example, a child pumping a swing. In the absence of damping, the system response grows unbounded. Linear resonance has been used in mechanisms for compressors, toothbrushes, shavers, and walking robots [6]. For more details on resonant robotic systems, we refer the reader to Ref. [7].

The requirements for a resonant mechanism for repeated tasks are proposed by Plooij and Wisse [8]: (1) the mechanism should not oppose the cyclic task, (2) it should smoothly transition from one operating point to another, (3) the system behavior at operating points should facilitate fast motion, and (4) within the operating range, an equilibrium point should exist when the potential energy reaches a minimum and the kinetic energy is a maximum.

It is important to note that in an experimental device when a system has uncertainties and/or nonlinearities, it is difficult to satisfy all four requirements. However, one can design a robust controller that will facilitate fulfilling the listed requirements.

Similar work using phase oscillators has been developed to create a method to estimate the state of the system. The state estimators are then used to create torque feedback to aid in elbow assistance [9–11].

Phase Oscillator Controller

A general equation for a mechanical system is described by a second order system with a given mass, m, dissipating term, b, and stiffness, k

$$m\ddot{x} + kx = -b\dot{x} \tag{1}$$

The frequency of oscillations of the system modeled by Eq. (1) is dependent on the mass, m, and the spring constant, k. The behavior of the amplitude of the oscillations is dependent on the damping coefficient or dissipating term, b. If b is greater than zero, the oscillations will shrink and disappear over time, and if b is less than zero, the oscillations will continue to grow. If b equals zero, the system has no damping, and the oscillations will remain at a constant amplitude creating a limit cycle. A desired controller for assisting repeating or recurring motion will (1) cancel the dissipating term and (2) adjust the size of the oscillations.

A "phase oscillator method" cancels the damping of the system to create a limit cycle and is based on the phase diagram of the system. The method adds energy to the system based on the phase angle, ϕ, shown in Fig. 1(*a*).

The phase angle can be used to determine when to add energy to the system and how much energy to add.

$$\omega = \sqrt{\frac{k}{m}} \tag{2}$$

$$\varphi = a\tan2\left(\frac{\dot{x}}{\omega}, x\right) \tag{3}$$

Consider a more general form of Eq. (1) as given by Eq. (4). This system has uncertain disturbance term, $u(t)$. In order to control uncertain dynamics, a forcing function proportional to the sine of the phase angle can be used to add energy to the system. The forcing function cancels the damping term and uncertainties allowing the system to oscillate at the natural frequency, ω, shown in Figs. 1(*b*) and 1(*c*). Equation 4 models the system with the phase oscillator

$$m\ddot{x} + kx + u(t) = -b\dot{x} + c\sin(\phi) \tag{4}$$

The limit cycle can be found by solving Eq. (4) analytically. Substituting Eq. (5) into Eq. (4) determines a sinusoidal solution given by Eq. (6). The amplitude, A, of the solution is given by Eq. (7).

$$\sin(\phi) = \frac{\left(\frac{\dot{x}}{\omega}\right)}{\sqrt{\left(\frac{\dot{x}}{\omega}\right)^2 + x^2}} \tag{5}$$

$$x(t) = A\sin(\omega t) \tag{6}$$

$$A = \frac{c}{b\omega} = \frac{c\sqrt{m}}{b\sqrt{k}} \tag{7}$$

Using the phase oscillator control method, the dissipating term is canceled, and the amplitude of the system can be changed directly by adjusting c. It can be shown that the system produces a globally stable limit cycle, see Fig. 2. A phase portrait was simulated in MAPLE software.

Applications of Phase Oscillator

The phase oscillator uses the velocity and displacement information to compute the phase that is used as a control signal to cancel the uncertain dynamics and dissipation. In other words, the phase oscillator attains the limit cycle at the natural frequency, driving the system to resonate. However, unlike an under damped system subjected to forcing at its natural frequency (i.e., linear resonance), the system response does not grow unbounded. Also, the system is vibrating in its natural mode; thus, the energy

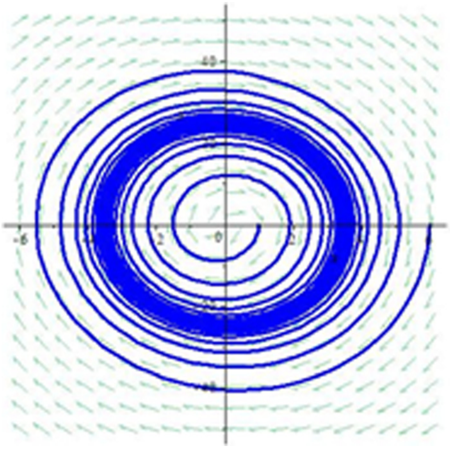


Fig. 2 A phase portrait of the system shown in Fig. 1 with the two initial conditions from Fig. 1. The limit cycle is globally stable. The horizontal axis is in units of position and the vertical axis is in units of velocity.

needed to sustain that mode is minimized. The controller does not act against the inertial effects.

In wearable robotic applications, unmodeled dynamics and uncertainty are introduced due to variation in mass, various ground contacts, boundary conditions, fatigue of the user, and shifting of the load, etc. Thus, a controller must be robust so that it can compensate for undesired effects. Metabolic savings are possible when the robotic system is not "fighting" the human activity instead tuning itself based on the state. One may use elegant nonlinear controllers, optimal and adaptive controllers, but the computational cost involved to calculate the control effort accurately is much higher. Thus, when implemented on an embedded system, these controllers may introduce "group delay." Because of this delay, the controller cannot run at a high update rate, apply accurate controls, or may not transition quickly. A few sample applications will be described next that use this control method. First, a simple hopping robot is described.

Hopping Robot

The hopper consists of an actuator and spring in parallel. The hopper is clamped to a ball bearing carriage and attached to a vertically mounted linear rail, Fig. 3(*a*). This configuration allows for free vertical motion.

The physical system is simulated using Eq. (8) and the parameters of the simulation are given as follows: mass of the hopper is 0.6 kg, spring rate is 2469.3 N/m, damping factor is 1.1 N/m s and the external applied force is 33 N

$$m\ddot{x} + b\dot{x} + (kx * e(x)) + mg = c\phi(t) \quad (8)$$

$$e(x) = 1, \quad \text{when} \quad x < 0 \quad \text{and} \quad 0 \quad \text{otherwise} \quad (9)$$

$$\phi(t) = 1 \quad \text{when} \quad 1.571 < \phi(t) < 1.588 \quad \text{and} \quad 0 \quad \text{otherwise} \quad (10)$$

Note that Eq. (8) is nonlinear contrary to Eq. (4). For experimental work, the frequency, ω, was chosen to be one and c was selected based on the desired hop height. An advantage of this controller is that natural frequency of the system does not need to be known. In our experimental work, we choose the frequency, ω, to be one. The phase angle in Eq. (8) is calculated using $a\tan2(\dot{x}, x)$.

The hopper is simulated and tested by dropping the system with the controller turned off to verify that the simulation parameters and physical system match.

Using the phase oscillator controller, the system achieves a steady state limit cycle independent of the initial conditions. Figure 3(*b*) shows the hopper reaching steady state from initial drops from 0 m, 0.1270 m, and 0.2376 m.

The controller is very robust to disturbances. One can bang on the top of the system or hold a wooden block against the system to limit hop height. In one example, Figs. 4(*a*) and 4(*b*), a hand blocks and disturbs the hop height. The system is disturbed, and returns back to a stable limit cycle.

Wearable Exoskeleton

Our goal is assist the legs to oscillate while walking and running by applying a torque at the hips based on a phase oscillator. Energy is added to the system and assists the limit cycle as the legs oscillate while walking and running.

The leg of the human body can be assumed to be a pendulum-like structure with inertia, damping, and stiffness [12–18]. To enhance the pendulum motion, a parametric excitation torque can be added. The direction of the torque must be switched at the correct timing and frequency and should be tuned with the frequency

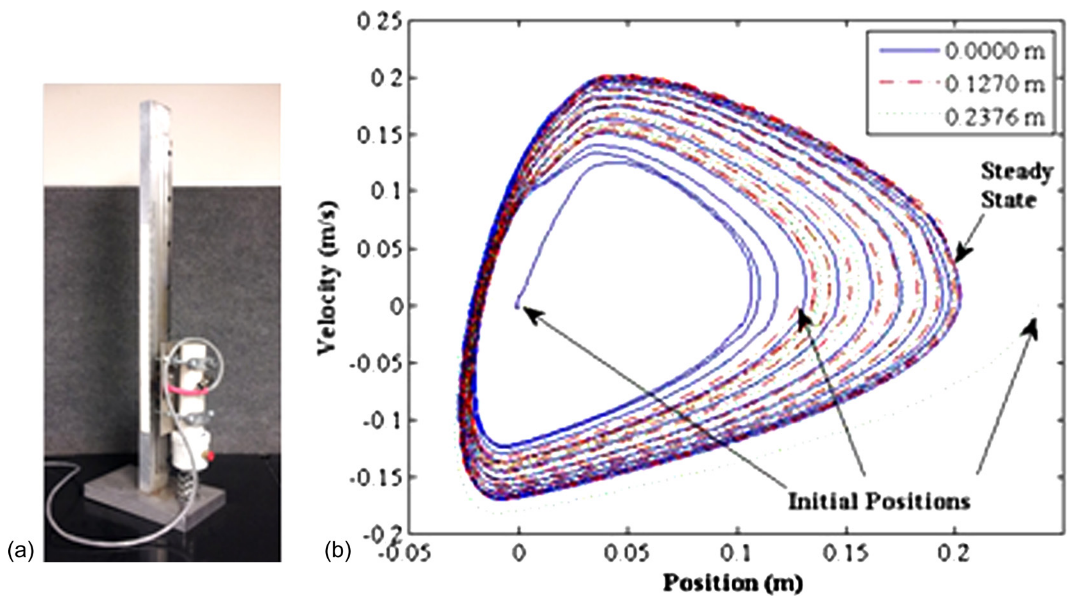


Fig. 3 (*a*) Robotic hopper assembly. (*b*) Phase portrait of hopper dropped from three different initial positions each achieving the same steady state limit cycle. The cycle is noncircular because there is a flight phase [5]. The hopper can interact with the ground or can enter a flight phase governed by drag and gravity.

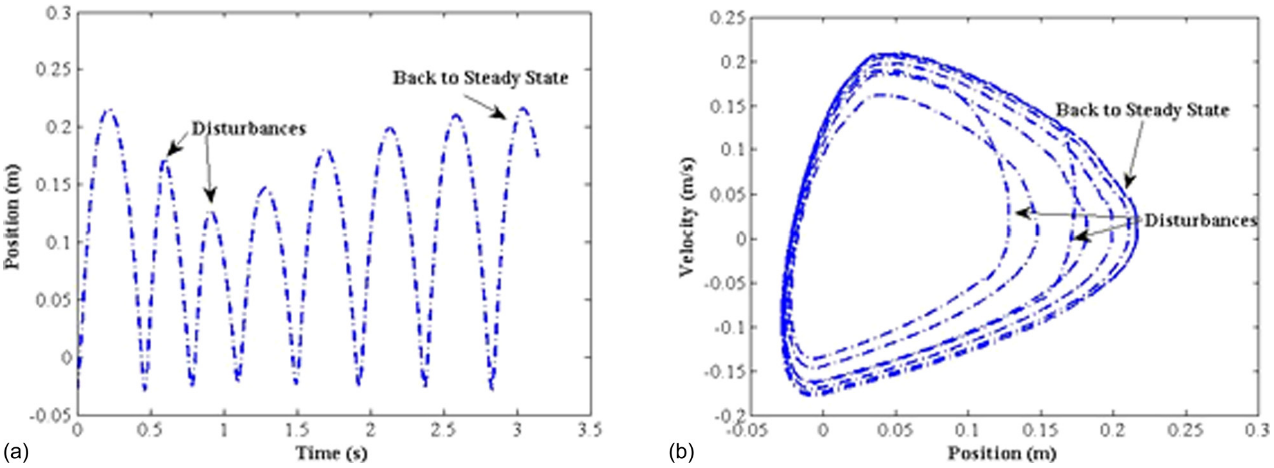


Fig. 4 (*a*) Actual position versus time of hopper experiencing two disturbances which limit height [5]. (*b*) Phase portrait of hopper experiencing two disturbances.

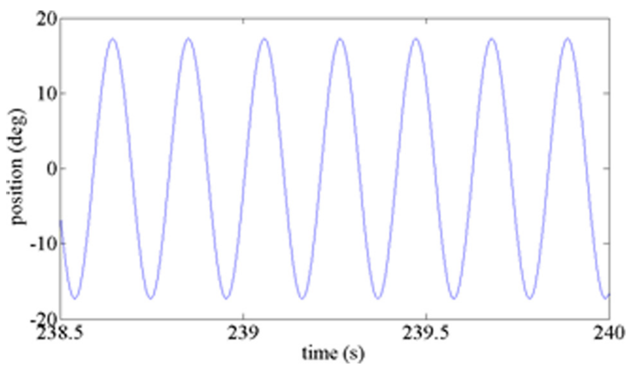


Fig. 5 The system oscillates back and forth at its natural frequency in a limit cycle. $I = 0.25$ kgm^2, $b = 0.007$ Nm/(rad/s), $k = 230$ Nm/rad, $c = 0.05$ Nm, initial velocity = (2/3)*pi rad/s, and initial position = (1/9)*pi rad.

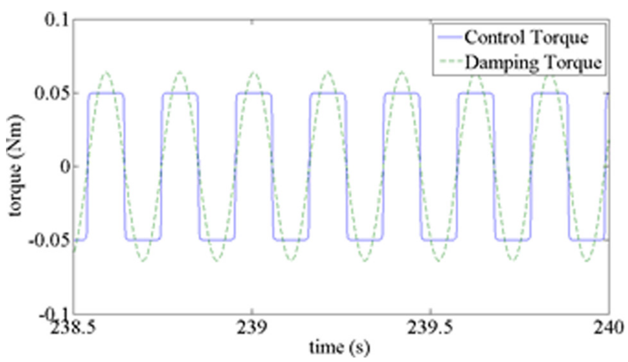


Fig. 6 The damping torque oscillates in a sine wave while the control torque generated by the sine of the phase angle behaves similar to a square wave. $I = 0.25$ kgm^2, $b = 0.007$ Nm/(rad/s), $k = 230$ Nm/rad, $c = 0.05$, initial velocity = (2/3)*pi rad/s, and initial position = (1/9)*pi rad.

of gait. A phase oscillating term, based on the phase portrait, determines the desired torque and adds positive power to the system.

A simulated system is modeled first which demonstrates that a control torque similar to a square wave can push and pull on a pendulum to create a limit cycle. This control signal is perfect for turning on and off pneumatic valves to actuate an air cylinder.

The analysis is shown for a rotational model: I represents the inertia, b represents the damping, k represents the rotational stiffness

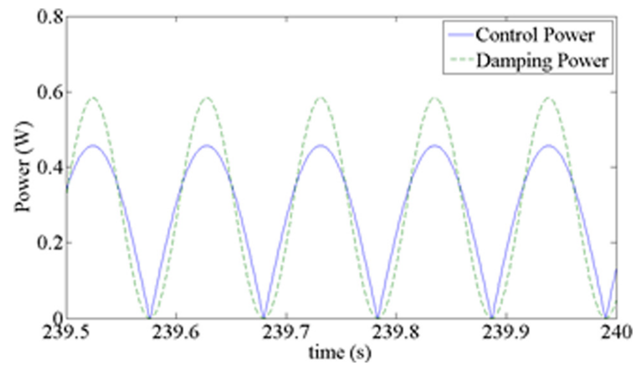


Fig. 7 The damping is shown positive instead of negative so it can be compared easily with the control power. The powers are similar but not equal. The energy for each curve over a cycle matches.

$$I\ddot{\theta} + b\dot{\theta} + k\theta = 0 \tag{11}$$

A forcing function is added using a phase oscillator

$$I\ddot{\theta} + b\dot{\theta} + k\theta = c\sin(\phi) = \frac{c\dot{\theta}}{\sqrt{\dot{\theta}^2 + \theta^2}} \tag{12}$$

If c is positive, the system oscillates back and forth. The energy is always bounded because as $\dot{\theta}$ gets large, in the limit the numerator and denominator cancel and just equal c. If c is negative, the energy is damped out and the system state goes to zero.

In a simulated example, a pendulum system with length 0.5 m, lumped mass of 1 kg, damping 0.007 Nm/(rad/s), and spring stiffness of 230 Nm/rad can be oscillated back and forth with a small excitation torque of 0.05 Nm. The limit cycle is robust and a wide range of initial conditions converge to the one limit cycle defined by the constant c. The system oscillates at its natural frequency of 4.83 Hz, see Fig. 5.

The damping of the system creates a torque to slow the system down while the torque created by the sine of the phase angle assists the movement of the system. Both signals are shown in Fig. 6, but the damping torque is shown with a positive sign instead of a negative sign to allow the signals to be plotted on top of each other.

The power into the system created by the control torque and the power out of the system created by the damping torque are similar but do not exactly match, see Fig. 7. However, if the power curves are integrated over a cycle, the energy into and out of the system

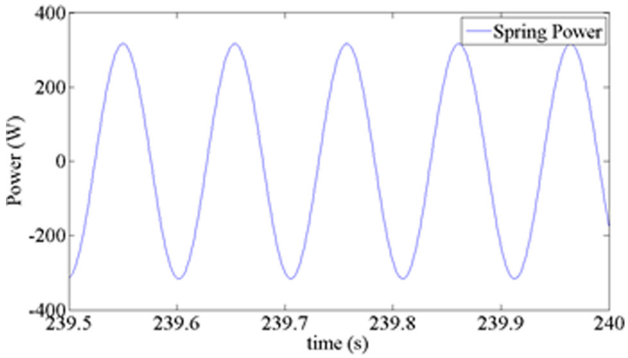


Fig. 8 The system is oscillating back and forth quickly and the spring torque is high resulting in large power oscillations in the spring

are equal and match. Even though the control power is quite low with peaks that are approximately 0.5 W, the power in the oscillating spring is quite high, 300 W, see Fig. 8. Note, the sign of the damping torque was changed to positive instead of negative so that the power curves could be plotted on top of each other. The spring power is quite high because the system is oscillating quickly and the spring stiffness is large. The spring stiffness was modeled based on the stiffness at the hip joint.

This control method is quite powerful because a low power oscillator can pump energy into the system and create large oscillations with high kinetic energy and high potential energy in the spring. The oscillations are bounded and create a limit cycle. We believe this method can be quite useful in developing wearable robots because small actuators (motors, pneumatic cylinders, hydraulic cylinders, etc.) can pump energy into the gait cycle or limb movement.

A powered exoskeleton has been designed and built to enhance walking and running, see Fig. 9(*a*). The control torque signal is used to trigger pneumatic valves to create torque at each hip joint. Torque assists flexion and extension of the thigh. The pneumatic cylinder pulls and pushes on the thigh plate attached to the leg. The linear force of the cylinder creates an assistive torque at the hip joint assisting flexion and extension.

One rate gyro is placed on each thigh above the knee joint to determine the angular velocity of each thigh, see Fig. 10(*a*). The time duration and motion for each gait cycle differ when comparing the left and right leg which makes it necessary for a separate

(a) (b)

Fig. 9 (*a*) A powered exoskeleton is used to assist the torque needed at the hips [2]. (*b*) A powered PogoSuit is used to assist the hopping motion. A pneumatic cylinder oscillates the small mass up and down in phase with the user [4].

sensor on each leg. The signal is integrated to determine the thigh angular position to determine the sine of the phase angle.

The phase angle, ϕ, of a person running on a treadmill in this exoskeleton is shown in Fig. 10(*b*). The left and right phase angles are periodic but are not exactly the same. The time that the right leg swings forward is slightly longer than the time for the left leg. As the leg waits for heel contact, the phase angle oscillates up and down for a brief 200–300 ms.

The control signals, seen as dashed lines in Fig. 10(*b*), are used to trigger valves. When the control signal is high, the valve is energized and the thigh is pushed forward. When the control signal is low, the spring inside the valve moves it into the second position to pull the leg back. Even on a treadmill, the gait motion is not perfectly the same.

The exoskeleton can seamlessly assist walking, running, and the transitions between the two. The seamless transitions are possible because a continuous control signal is generated and used as a triggering mechanism. We measure the phase angle at 500 Hz and trigger the system in phase with the subject whether they are walking slow or fast.

The exoskeleton, in pilot work, has remarkably shown reduced metabolic cost comparing the device worn to no device. The difficulty for an exoskeleton to show reduced metabolic cost is that it must first overcome the metabolic burden of carrying additional weight and, second, must add energy at the correct time. The total system weight is 11.6 lbs or 5.26 kg [2].

At the Army Research Laboratory, we demonstrated the device on two subjects. On the first subject, the device was too big and an increase in metabolic cost was shown at the faster speed. On the second subject, a tall male, metabolic savings was shown, see Table 1. The values shown in the table in bold correspond to metabolic savings. The user had reduced metabolic cost comparing device powered ON versus no device at all. Testing was performed at ARL on Sept. 26–27, 2013. All testing was performed under a protocol approved by an institutional review board.

PogoSuit

The goal is to oscillate a secondary mass in phase with the user's walking and running to reduce the metabolic cost of gait.

The secondary mass is accelerated up and down by a motor or a pneumatic cylinder and the reaction force to hold the actuator in place creates the external force. If one runs with a nonmotorized backpack, the weight oscillates up and down with incorrect phase and hinders the running motion. On the other hand, if the weight is oscillated based on the phase angle in the phase portrait, the running motion is enhanced due to energy pumping.

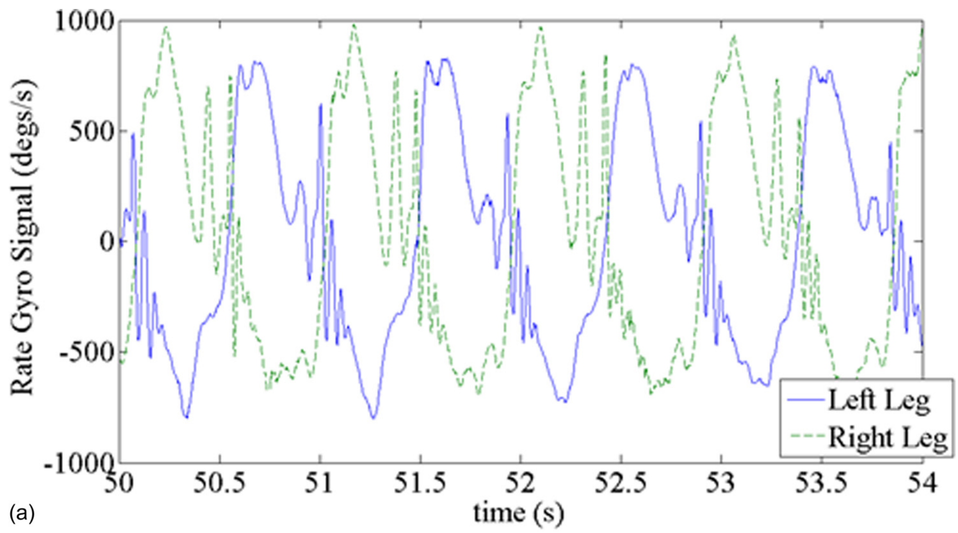
A small oscillating mass adds small, positive power to the hopping motion enhancing hop height response. In a counter example, if the oscillating mass is moved in an antiphase motion, the hop height is decreased and a resistance training device or "absorber/brake" is created.

A powered PogoSuit has been designed and built to enhance hopping, see Fig. 9(*b*). One accelerometer is placed at the waist above the waist belt to determine the vertical acceleration of the trunk. The signal is integrated to determine the trunk velocity and integrated a second time to determine position in order to calculate the sine of the phase angle.

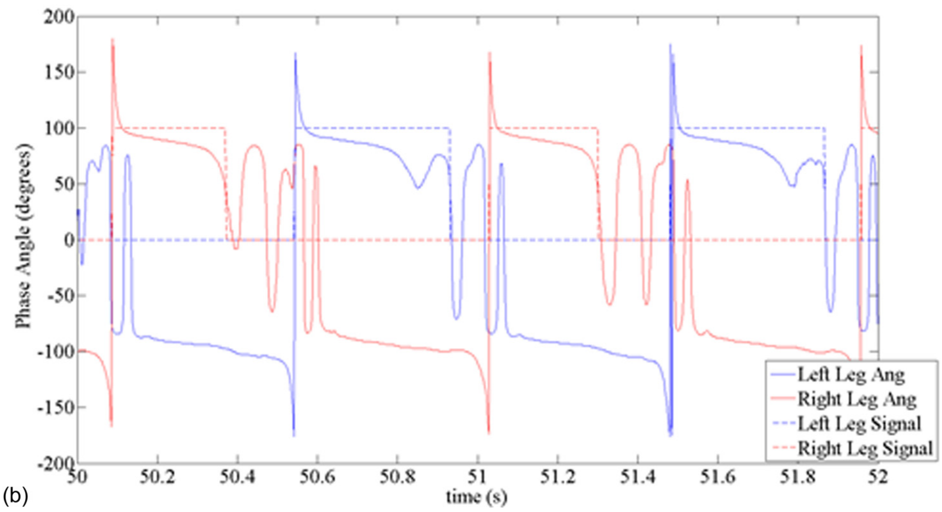
In demonstrations, with the device in phase, hop height has been increased and metabolic cost has been decreased. On the other hand, with the device in antiphase motion the metabolic cost is greatly increased [4].

Energy Harvesting

Energy harvesting systems are typically mass, spring, damper systems that have a tuned natural frequency to match the frequency of the input signal. However, these systems do not harvest much energy if the input frequency varies slightly from the tuned



(a)



(b)

Fig. 10 (*a*) A rate gyro is mounted on each thigh. The signals and time durations for each leg differ [2]. (*b*) The phase angle for the left and right leg. The left and right control signals are shown in dashed lines [2]. The phase angle for the left and right leg is measured at 500 Hz in the microprocessor. As the leg swings forward, the control signal turns high and as the leg swings backward, the control signal turns low.

harvester's natural frequency. In comparison, a tuned phase oscillator can harvest more energy if the frequency varies slightly.

Given a known oscillatory source like ocean waves, wind or human walking

$$\text{input} = A\cos(\omega t) = 2.5\cos(\omega t) \tag{13}$$

A generator is modeled as a linear damping element, b [19]. A simulated model is tuned to harvest energy with a constraint on the output movement to be ± 0.5 m.

$$m\ddot{x} + b\dot{x} + kx = A\cos(\omega t) \tag{14}$$

$$2\ddot{x} + 0.5\dot{x} + 200x = 2.5\cos(\omega t) \tag{15}$$

Table 1 Mean (SD) metabolic cost when running with powered hips in Fig. 9(*a*) on a treadmill at ARL

Test	Metabolic cost (ml/min kg)
Subject 1, small female (mass = 59.1 kg and ht = 162 cm)	
1. Running at 5.5 mph on a treadmill, no device	26.52 (3.7)
2. Running at 5.5 mph on a treadmill with powered hip system ON	26.48 (4.0) (−0.2%, no difference)
3. Running at 6.5 mph on a treadmill, no device	29.4 (2.7)
4. Running at 6.5 mph on a treadmill with powered hip system ON	31.3 (3.5) (6.7% increase)
Subject 1, tall male (mass = 66.1 kg and ht = 181.8 cm)	
5. Running at 6 mph on a treadmill, no device	31.6 (2.8)
6. Running at 6 mph on a treadmill with powered hip system ON	**29.1 (3.4) (8.0% savings)**
7. Running at 8 mph on a treadmill, no device	40.8 (2.4)
8. Running at 8 mph on a treadmill with powered hip system ON	**36.6 (1.7) (10.2% savings)**

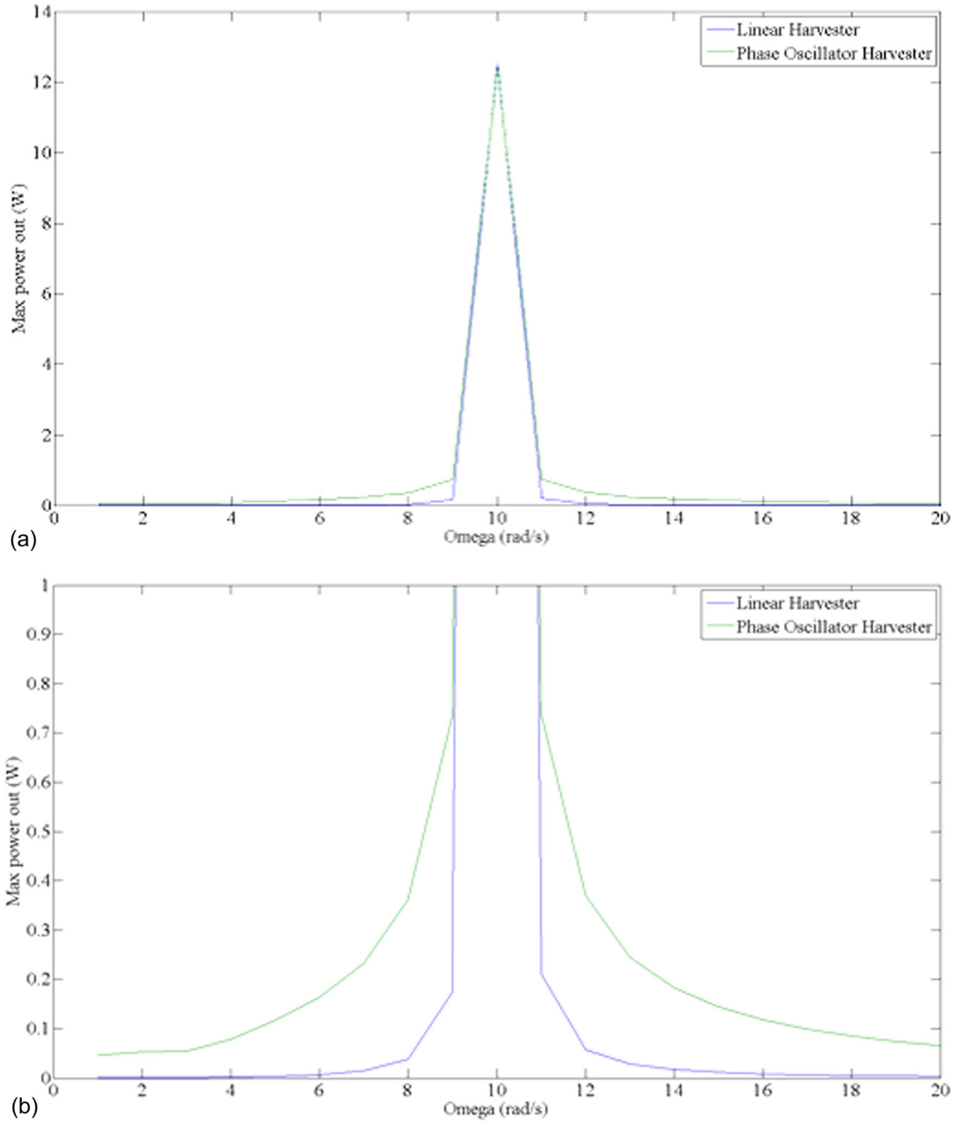


Fig. 11 At the natural frequency of the system, 10 rad/s, both the linear oscillator generator and the phase oscillator harvester can produce 12.5 W. The simulated measurement was taken at steady state with a constraint of 0.5 m oscillations. At 9 rad/s, the linear generator can produce 0.17 W while the nonlinear generator can produce 0.73 W. (0.04 W versus 0.36 W at 8 rad/s).

If ω is varied, the power that can be harvested varies dramatically shown in Fig. 11.

In comparison, given a nonlinear system, a generator is modeled as a nonlinear damping element based on the phase oscillator, c. A second model is tuned to harvest energy with the same constraint on the output movement. A linear damping term, d, was added to limit the output movement. Because the phase oscillator does not inhibit high speed, large amplitudes can be achieved. In fact, the system oscillations will continue to grow at longer time periods unless the linear damping term is added.

$$m\ddot{x} + \frac{\frac{c}{\omega}}{\sqrt{\frac{\dot{x}^2}{\omega^2} + x^2}}\dot{x} + d\dot{x} + kx = A\cos(\omega t) \tag{16}$$

$$2\ddot{x} + \frac{0.2}{\sqrt{\frac{\dot{x}^2}{100} + x^2}}\dot{x} + 0.1\dot{x} + 200x = 2.5\cos(\omega t) \tag{17}$$

Again, if ω is varied, the power that can be harvested varies dramatically as shown in Fig. 11. The phase oscillator performs much better when the frequency of oscillation is not matched to the natural frequency of the system. On the other hand, the natural frequency of a system can be tuned using a JackSpring actuator [20].

Limit Cycles

Other robotics researchers have been studying how to assist the gait of a walking robot using energy conservative limit cycles [21]. We believe these methods are powerful because autonomous robotic systems need energy efficient and robust controllers to create natural gait patterns.

Conclusions

Our goal has been to develop a method to assist the oscillating or cyclic motion while walking and running. We developed a phase oscillator to develop a torque signal that is bounded and creates a limit cycle. The torque signal is small, oscillatory in nature, and cancels out the damping in the system. The signal is continuous and is used to trigger the actuators in our hip exoskeleton. It assists walking and running and easily transitions from walking to running and back again. In demonstration work, the exoskeleton is very promising showing metabolic augmentation. We have shown that this controller is very general and it can be used to create a hopping robot, a PogoSuit system that oscillates a secondary mass to pump energy into the user, and an energy harvesting method that is robust to changing frequencies.

Acknowledgment

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Appendix

Canonical form of the phase oscillator where c defines the amplitude of the oscillation

$$\ddot{x} + 2\zeta\dot{x} + x = \frac{c}{\sqrt{\dot{x}^2 + x^2}}\dot{x} \tag{A1}$$

Canonical form of the Van der Pol oscillator

$$\ddot{x} + \mu x^2\dot{x} + x = \mu\dot{x} \tag{A2}$$

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